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Observation of volume variation effects in turbulent free convection

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Abstract

This paper presents preliminary results of laboratory experiments designed to study the effects of specific volume variation as regards the basic physical mechanisms governing weakly compressible flows. Turbulent free convection developing in a large vertical open tunnel was investigated. Results were analyzed in the framework of an original formulation of the Navier–Stokes equations in which no simplifying assumptions regarding density variations were made, and where the terms representing specific volume *fluctuation* effects could be isolated and examined separately. Non-isovolume mechanisms were found to play a noticeable role in the transport equations of internal and turbulent energy, and had to be accounted for in budget analysis. In contrast, the analysis showed that the form of the transport equation of mean momentum obtained and used classically when the Boussinesq approximation applies could still be used in the present case, consistently with earlier observations in the atmosphere. Results put forward that the role played by non-isovolume mechanisms depends strongly on the mean velocity divergence and, with respect to the temperature field, on the dissipation rate of the temperature variance.

1. Introduction

Considerations will here be restricted to fluids heated from below for which buoyancy effects form the only source of motion. The emphasis will be put on large-sized flows whose mean temperature differs only slightly from ambient temperature. This will be referred to as turbulent free convection.

Although turbulent free convection is a rather common regime for flows in the atmosphere as well as in various industrial processes, our understanding of the dynamics of buoyant turbulent motion still remains incomplete. One of the open questions in this field is the role of density variations as regards the basic physical mechanisms governing turbulent free convection.

Turbulent free convection continues to be analyzed and described most often using a quasi-incompressible formulation of the Navier–Stokes equations, as can be seen from the different contributions in [1]. Furthermore, numerical simulations often use models and approximations that have been established from experiments whose analysis itself was based on these same models and approximations, and where the non-measurable terms were deduced by using already simplified closures. Results from such experiments are to some extent necessarily biased and, as a general rule, such an approach cannot provide an understanding of terms that have been ignored in the formulation of the equations used for analysis. If one has need for closures for a model of convection, then one needs to obtain this information from studies of convection with as few hypotheses as possible for data analysis.

Therefore, laboratory experiments were conducted in a large open vertical tunnel with air as working fluid so as to

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Nomenclature

A	generalized effective density of volume source strength in the balance equation for Ψ
C_v	specific heat of air at constant volume
g	gravitational acceleration
J	generalized effective density of surface source strength (molecular flux) in the balance equation for Ψ
j	fluctuating part of J
P	pressure
p	pressure fluctuation
$\vec{Q}$	conductive heat flux
$\vec{q}$	fluctuating part of $\vec{Q}$
R	gas constant for air
Rad	radiation
Rad′	fluctuating part of Rad
t	time
T	temperature
U	specific volume of air
$\vec{U}$	velocity vector
U, V, W	velocity components
$\vec{u}$	fluctuating part of $\vec{U}$
u, v, w	fluctuating velocity components
U_θ	convection velocity of temperature scales
x, y, z	co-ordinates (the vertical co-ordinate x designates also the mean flow direction)

Greek symbols

α	thermal diffusivity
γ	ratio of air specific heats ($= C_p/C_v$)
ε_u	dissipation rate of turbulent kinetic energy
ε_θ	dissipation rate of the temperature variance
Φ	dissipation function
κ	turbulent kinetic energy
Π_ψ	generalized deviation term representing volume fluctuation effects of second order
Π_θ	Π_ψ-term in the transport equation of internal energy
$\vec{\Pi}_u$	$0\Pi_\psi$-term in the transport equation of mean momentum
ν	kinematic viscosity
Σ, Σ_{ij}	stress tensor
σ, σ_{ij}	fluctuating part of the stress tensor
θ	temperature fluctuation
τ, τ_{ij}	rate-of-strain tensor (= deviatoric stress tensor)
ϑ	fluctuating part of U
Ψ	intensive quantity equal to the amount of any extensive fluid property (e.g., mass, momentum, kinetic energy, etc.) per unit mass of fluid; here Ψ may be any scalar-, vector- or tensor-valued function of time and position
ψ	fluctuating part of Ψ

Superscripts

—	time and surface average
=	tensor

Subscripts

a	ambient fluid
k	k component

Other notations

$\mathrm{D}/\mathrm{D}t$	time derivative following mean motion ($= \partial/\partial t + \overline{\vec{U}} \cdot \nabla$)
$\vert_x$	projected component along the vertical axis

investigate density variation effects in unsteady turbulent free convection. The temperature difference between the flow and the ambient fluid outside the tunnel was approximately 10 K. This arrangement corresponded to the typical case of application of the Boussinesq or anelastic approximations. However, the Rayleigh number characteristic of this flow configuration was of the order of 10^{11} and thus was typical of flows for which, like strongly heated jets, compressibility effects have to be fully accounted for in the transport equations.

Results were analyzed using the alternative formulation of the Navier–Stokes equations proposed by Rey [2], for weakly compressible flows. No a priori simplifying assumptions were made in the equations as regards dilatation mechanisms.

Part of the results regarding the conservation of mean momentum, internal energy and the temperature and velocity variances are presented and discussed in this paper. Much more information was gained out of this work through the examination of the balance of the turbulent heat flux transport equation, and through spectral analysis. This provided an opportunity for discussions regarding the mass-weighted averaging approach of Favre [3]. Relevant findings have been fully reported in [4]. The present paper is rather intended to communicate about the spirit and the potential benefits of the approach taken throughout this work, and about the questions this study raised.

2. Description of the experimental apparatus

2.1. Set-up and working principle

The experimental facility designed for this study was a 10-m high vertical open tunnel with a square cross-section of constant dimensions 3 m × 3 m (Fig. 1). The flow was generated using an electrically heated grid consisting of a perpendicular arrangement of two sets of 29 resistive rods 15 mm in diameter. The grid mesh size was 10 cm. The flow generated in this manner was so that buoyancy was the sole source of motion, and production of turbulent kinetic energy by mean shear was negligible compared to production by buoyancy. During experiments, all the resistive elements were switched to full power so as to ensure constant and steady heating conditions. This corresponded to a total electrical power supply of 80 kW.

Simultaneous measurements of fluctuating velocity and temperature were performed by scanning multi-wire probes within horizontal planes at various test sections located 2, 3, 4, 5 and 6 m downstream of the heated grid. Thus, measurements were taken in a region where the flow could be considered fully developed, and where the turbulence could be reasonably assumed to be homogeneous and isotropic. More details on measurement procedures are given in [4].

2.2. Flow special peculiarities

As measurement sessions lasted about 5 h, our ability to achieve experiments with ambient temperature variations at a steady rate was severely limited. The continual heat discharge into the laboratory hall caused an increase in the ambient mean temperature at a rate dependent on the heat supply and the rate of heat exchange between the building and outside. Moreover, building losses were influenced by the diurnal meteorological cycle. The mean flow structure inside the vertical tunnel in turn was affected by the ambient temperature changes through re-entrainment of ambient air at the tunnel entry section and through altered blockage effect at the exit section.

Therefore, a monitoring system was set up providing on-line information about the ambient and flow temperature stratification during experiments to allow for the temporal evolution of mean temperatures in the data analysis. The apparatus consisted of 23 thermocouples. Of those, 15 were hung in the laboratory hall along 3 ropes stretched vertically between the building floor and the ceiling, at 3–10-m distance from the convection tower. The 7 remaining thermocouples were fixed at 2, 3, 4, 5, 6, 8 and 10 m above the grid plane along a fourth rope located inside the tunnel.

Mass experiments were carried out to estimate convection velocities of the temperature and kinematic structures because of the breakdown of Taylor's hypothesis regarding temperature scales. Convection velocities were determined from space–time correlation analysis. Longitudinal space–time correlations could not

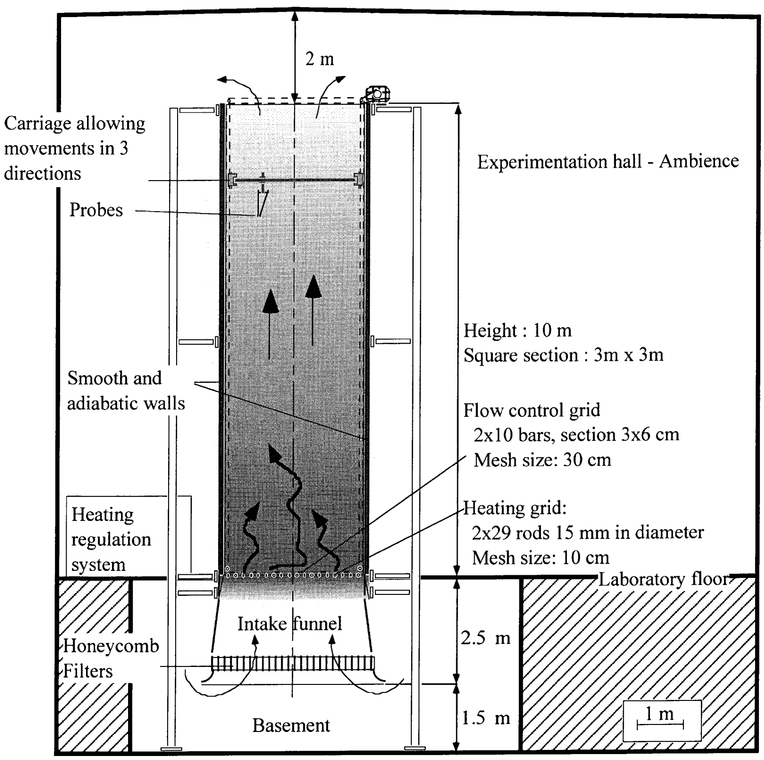

Fig. 1. Schematic view of the experimental apparatus.

be measured directly because of wake interference between the probes. Therefore, these were inferred from measurements of transversal and angular space–time correlations. The convection velocity U_θ was found to be constant and equal to $1.5U$ within the investigated flow region. That of the kinematic structures turned out to be equal to U. This had to be accounted for in the calculation of ε_θ and ε_u involving the determination of the Taylor's micro-scales from the relevant dissipation spectra. Note that the ratio U_θ/U could be as high as 5 immediately downstream of the grids. Hence, for $U_\theta = 5U$, the dissipation rate of the temperature variance would have been overestimated by a factor of 25 if in the calculation one had used U instead of the appropriate convection velocity.

3. An alternative set of equations

3.1. Philosophy

For compressible fluids, it seems consistent to use a mass formulation. Therefore, experimental results were analyzed in the framework of a new formulation of the Navier–Stokes equations derived from a local balance analysis for elementary mass of fluid. This is the essence of the approach proposed by Rey in [2] that is briefly outlined in the following. Here the so-called 'volume formulation', which consists in deriving all necessary equations from local balance analysis for elementary volume of fluid will be referred to as the 'classical formulation' or 'classical approach'.

For any arbitrary extensive quantity associated with the fluid (e.g., mass, momentum, kinetic energy, etc.), whose amount per unit mass of fluid is denoted by Ψ, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_M \Psi\,\mathrm{d}m = \int_M A\,\mathrm{d}m + \int_F (J\vec{n})\,\mathrm{d}s.$$

Here dm denotes that integration is to be performed over the control mass M of fluid. By F, we denote the closed bounding surface of M. The symbols A and J represent the effective densities per unit mass and per unit surface area of source/sink strength, respectively. By interchanging differentiation and integration in the left-hand side of the equation above since M is constant, and by using Green's transformation in the right-hand side, it becomes

$$\int_M \left(\frac{\mathrm{d}\Psi}{\mathrm{d}t} - A - \mathsf{U}\,\mathrm{div}(J)\right)\mathrm{d}m = 0.$$

Note that, in this approach, we are considering integration over a material body that is not function of time by definition, while the classical approach would have looked at the volume D containing M, with D a function of time when specific volume (or density alternatively) is variable.

But the last equation is true for any material body of or any portion of material body. Therefore, the integrand itself must be identically zero so that

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} - A - \mathsf{U}\,\mathrm{div}(J) = 0.$$

Subsequently, by splitting up $\vec{U}$, U, Ψ, and J into their mean and centered fluctuating components with respect to Reynolds statistical decomposition we obtain successively:

$$\begin{aligned}
&\frac{\mathrm{d}\Psi}{\mathrm{d}t} + (\overline{\vec{U}} + \vec{u})\,\mathrm{grad}(\overline{\Psi} + \psi) \\
&\quad - A - \overline{\mathsf{U}}\,\mathrm{div}(J) - \vartheta\,\mathrm{div}(\overline{J} + j) = 0,\\
&\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \overline{\vec{U}}\,\mathrm{grad}(\Psi) + \vec{u}\,\mathrm{grad}(\overline{\Psi} + \psi) \\
&\quad - A - \overline{\mathsf{U}}\,\mathrm{div}(J) - \vartheta\,\mathrm{div}(\overline{J} + j) = 0,\\
&\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \overline{\vec{U}}\,\mathrm{grad}(\Psi) + \vec{u}\,\mathrm{grad}(\overline{\Psi}) + \vec{u}\,\mathrm{grad}(\psi) \\
&\quad + \psi\,\mathrm{div}(\vec{u}) - A - \overline{\mathsf{U}}\,\mathrm{div}(J) \\
&\qquad = \vartheta\,\mathrm{div}(\overline{J}) \quad + \vartheta\,\mathrm{div}(j) + \psi\,\mathrm{div}(\vec{u}),\\
&\frac{\mathrm{D}\Psi}{\mathrm{D}t} + \vec{u}\,\mathrm{grad}(\overline{\Psi}) + \mathrm{div}(\psi\vec{u}) - A - \overline{\mathsf{U}}\,\mathrm{div}(J) \\
&\quad = \vartheta\,\mathrm{div}(\overline{J}) + \underbrace{\vartheta\,\mathrm{div}(j) + \psi\,\mathrm{div}(\vec{u})}_{\Pi_\psi}.
\end{aligned} \tag{1}$$

Eq. (1) is the generalized formulation of the local balance equation for Ψ. Interestingly enough, this equation can be written schematically using the diagram below:

$$\left\{\begin{array}{l}\text{Extension of the balance equation}\\ \text{for an incompressible fluid allowing}\\ \text{variations of specific volume with space}\\ \text{and time, but } \textit{precluding fluctuations}\\ \text{of specific volume}\end{array}\right\}$$
$$= \underbrace{\left\{\begin{array}{l}\text{Interactions}\\ \text{between}\\ \text{volume}\\ \text{fluctuation}\\ \text{and mean}\\ \text{fields}\end{array}\right\} + \left\{\begin{array}{l}\text{Pure volume}\\ \text{variation through}\\ \text{interactions between}\\ \text{volume fluctuation}\\ \text{and divergence}\\ \text{of the velocity}\\ \text{fluctuation}\end{array}\right\}}_{\text{deviation}}.$$

The terms on the right-hand side of Eq. (1) are related to physical mechanisms associated with solely the *fluctuation* of specific volume. These terms have been called *deviation terms* for they can be considered representative of the deviation from a situation for which specific volume would vary, but not fluctuate. Finally, note that the statistical decomposition of A in Eq. (1) may introduce additional deviation terms when, e.g., it is proportional to the divergence of velocity as is the case in the budget equation of internal energy (see further).

This formulation is interesting in many regards. It provides a more general framework for the study of a broad range of flow configurations in turbulent free convection since, so far, no simplifications have been introduced. Physical mechanisms are viewed from a different perspective in comparison with the classical approach. In particular, this formulation allows a better identification of the mechanisms related to volume variation effects (see also Section 3.2). Some other advantages of this formulation are worth being put forward immediately:

- The left-hand side of Eq. (1) corresponds to the nearly incompressible case for which variations of $\overline{\mathrm{U}}$ are allowed (instead of variations of $\overline{\rho}$ in the classical approach). Interestingly enough, the presence of $\overline{\mathrm{U}}$ in the left-hand side of Eq. (1) will not introduce correlations of higher order than in the classical formulation. Consequently, it may be reasonable to propose models of the statistical correlations involved in Eq. (1) by applying model structures similar to those used in the classical formulation.
- All terms on the right-hand side of Eq. (1) are identically zero when specific volume does not *fluctuate* (which a priori is different from does not *vary*).
- The right-hand side of Eq. (1) allows a distinction between specific volume fluctuation mechanisms due to (i) correlations with specific volume fluctuations or (ii) correlations with the divergence of velocity fluctuations.
- The terms on the right-hand side of Eq. (1) are 'new' in that sense they do not appear explicitly in the volume formulation. Their physical interpretation however is clear. One objective of the work presented here was to estimate the contribution of these terms in the case of turbulent free convection where they are usually ignored (especially the correlation with $\mathrm{div}(\vec{u})$).

Eq. (1) must be compared to the equation obtained by the classical approach that relies on local balance analysis for elementary volume of fluid:

$$\frac{\mathrm{D}\Psi}{\mathrm{D}t} + \mathrm{div}(\Psi\vec{u}) - A - \frac{1}{\overline{\rho}}\,\mathrm{div}(J) = \frac{\rho'}{\rho\overline{\rho}}\,\mathrm{div}(J) + \Psi\,\mathrm{div}(\vec{u}).$$

The equation above represents the same physics as Eq. (1). Only the way density variation effects are formally distinguished through this classical formulation does not exactly coincide with the way specific volume variation mechanisms are made explicit in the mass formulation. This is a direct consequence of the fact that $\overline{\mathrm{U}} = \overline{1/\rho} \neq 1/\overline{\rho}$, among others. Similarly, the leftmost term on the right-hand side of the equation above introduces a difficulty from the viewpoint of an experimenter. The first-order expansion series required to make this term tractable, because it is related to ρ in instantaneous value, will automatically increase the degree of the resulting correlation. This is not the case with Eq. (1), as is shown in Section 3.6.

3.2. Hierarchy of terms

Eq. (1) has been written so that, from left to right, a certain hierarchy is respected regarding the effects of volume variation. Thus, the left-hand side of Eq. (1) encompasses terms usually held to be dominant in the balance equations. These terms are to be associated with 'macroscopic' variations of volume. The leftmost term on the right-hand side expresses couplings between fluctuations of specific volume and mean local surface interactions between the selected mass of fluid and the surrounding matter. The contribution of this coupling to turbulence production may be noticeable in turbulent free convection if not dominant in some particular cases. The term Π_ψ which contains only terms whose mean value is non-zero, puts together the deviation terms considered, a priori, negligible compared to the other terms of the budget because they reflect volume fluctuation effects of second order. The rightmost term in Π_ψ was written last since the coupling with the fluctuation of specific volume (in the form of the divergence of the velocity vector) it represents, is usually assumed to play no significant role in the balance.

The consistency of the hierarchy described above may be better understood when considering the following classes of fluids:

- For an incompressible fluid, i.e., whose density or specific volume does not vary in space nor in time, the deviation terms are identically zero. Eq. (1) reduces to the generalized transport equation for incompressible fluids.
- For weak fluctuations of specific volume, the term Π_ψ may remain negligible in the balance. Eq. (1) takes a form close to the Boussinesq formulation whereby the coupling $\vartheta\,\mathrm{div}(\overline{J})$ will operate only in the balance of the transport equation of ψ and of correlations with ψ, but will not play any role as regards the balance of the transport equation of $\overline{\Psi}$.
- For strongly compressible flows, Π_ψ has to be fully accounted for in the transport equation of Ψ and, subsequently, in all transport equations of correlations with the fluctuation ψ.

In the two last cases, one must pay attention to the term $\vartheta\,\mathrm{div}(\overline{J})$, which yields a source term proportional to $\overline{\vartheta\psi}$ or $\overline{\vartheta\vec{u}}$ in the transport equations of the variance and the turbulent flux of ψ, respectively. The contribution of those correlations to the balance will be all the more noticeable as $\mathrm{div}(\overline{J})$ itself is significant, which is not something that is obvious to predict.

The contribution of the deviation terms to the balance of the transport equations for mean momentum, internal energy, the variances of temperature and velocity, and the streamwise turbulent heat flux was examined. Only part of the relevant results is discussed in this paper. We give below two applications of Eq. (1) and the transport equations used for data analysis.

3.3. Application of Eq. (1) to the local transport equation of momentum

$$\underbrace{\frac{\mathrm{D}\vec{U}}{\mathrm{D}t} - \vec{g} - \overline{\mathrm{U}}\,\mathrm{div}(\Sigma) + \vec{u}\cdot(\overline{\overline{\mathrm{grad}}\,\vec{U}}) + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}})}_{\text{balance without volume fluctuation}} = \underbrace{\vartheta\,\mathrm{div}(\Sigma) + \vec{\Pi}_u}_{\text{deviation}} \tag{2a}$$

with

$$\vec{\Pi}_u = \vartheta\,\mathrm{div}(\sigma) + \vec{u}(\vec{\nabla}\cdot\vec{u}). \tag{2b}$$

3.4. Application of Eq. (1) to the local transport equation of internal energy

$$\underbrace{\frac{\mathrm{D}(C_{\mathrm{v}}T)}{\mathrm{D}t} + RT\vec{\nabla}\cdot\overline{\vec{U}} + \overline{\mathrm{U}}(\vec{\nabla}\cdot\vec{Q} - \mathrm{Rad} - \Phi) + \vec{u}\cdot\vec{\nabla}(C_{\mathrm{v}}\overline{T}) + \vec{\nabla}\cdot(C_{\mathrm{v}}\theta\vec{u})}_{\text{balance without volume fluctuation}} = \underbrace{\vartheta(-\vec{\nabla}\cdot\overline{\vec{Q}} + \mathrm{Rad} + \Phi) + \Pi_\theta}_{\text{deviation}} \tag{3a}$$

with

$$\Pi_\theta = -\vartheta(\vec{\nabla}\cdot\vec{q} - \mathrm{Rad}') - (R\overline{T} + (R - C_{\mathrm{v}})\theta)\vec{\nabla}\cdot\vec{u}. \tag{3b}$$

3.5. Practical transport equations used for data analysis

3.5.1. Momentum

Eq. (2a) yields for mean momentum after averaging

$$\frac{\mathrm{D}\overline{\vec{U}}}{\mathrm{D}t} + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}}) - \vec{g} - \overline{\mathrm{U}}\,\mathrm{div}(\overline{\Sigma}) = \overline{\vec{\Pi}}_u. \tag{4a}$$

Note that buoyancy forces can be made explicit in the equation above by considering the mean field to deviate from an arbitrary aerostatic and adiabatic field at rest. We can thus write for the transport equations of the mean and fluctuating parts of momentum, respectively:

$$\frac{\mathrm{D}\overline{\vec{U}}}{\mathrm{D}t} + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}}) + \left(\frac{\overline{\mathrm{U}} - \mathrm{U}_0}{\mathrm{U}_0}\right)\vec{g} - \overline{\mathrm{U}}(\mathrm{div}(\overline{\tau}) - \vec{\nabla}(P - P_0)) = \overline{\vec{\Pi}}_u,$$

$$\frac{\mathrm{D}\vec{u}}{\mathrm{D}t} + \vec{u}\,\overline{\overline{\mathrm{grad}}}(\overline{\vec{U}}) + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}} - \overline{\overline{\vec{u}\otimes\vec{u}}}) - \overline{\mathrm{U}}\,\mathrm{div}(\sigma) = \vartheta\left(\mathrm{div}(\overline{\tau}) - \vec{\nabla}(P - P_0) - \frac{\vec{g}}{\mathrm{U}_0}\right) + \vec{\Pi}_u - \overline{\vec{\Pi}}_u.$$

However, local specific volume fluctuations may yet play an identifiable role on the structure of turbulent convection. Buoyant motion of a parcel of fluid, and associated energy mechanisms, indeed are determined by the interactions with the immediately surrounding fluid particles, but not by the ambient fluid state or any arbitrary reference state. Therefore, we chose the mean temperature and velocity fields themselves as a reference state instead of any other arbitrary reference field. [2] We thus used

$$\frac{\mathrm{D}\vec{u}}{\mathrm{D}t} + \vec{u}\,\overline{\overline{\mathrm{grad}}}(\overline{\vec{U}}) + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}} - \overline{\overline{\vec{u}\otimes\vec{u}}}) - \overline{\mathrm{U}}\,\mathrm{div}(\sigma) = \frac{\vartheta}{\overline{\mathrm{U}}}(\overline{\vec{\mathrm{D}}}_u - \vec{g} - \overline{\vec{\Pi}}_u) + \vec{\Pi}_u - \overline{\vec{\Pi}}_u \tag{4b}$$

with

$$\overline{\vec{\mathrm{D}}}_u = \frac{\mathrm{D}\overline{\vec{U}}}{\mathrm{D}t} + \mathrm{div}(\overline{\overline{\vec{u}\otimes\vec{u}}}).$$

3.5.2. Internal energy

We used for the mean

$$\frac{\mathrm{D}(C_{\mathrm{v}}\overline{T})}{\mathrm{D}t} + \vec{\nabla}\cdot(C_{\mathrm{v}}\overline{\theta\vec{u}}) + R\overline{T}\vec{\nabla}\cdot\overline{\vec{U}} + \overline{\mathrm{U}}(\vec{\nabla}\cdot\overline{\vec{Q}} - \overline{\mathrm{Rad}} - \Phi) = \overline{\Pi}_\theta \tag{5a}$$

and for the fluctuation

$$\frac{\mathrm{D}(C_{\mathrm{v}}\theta)}{\mathrm{D}t} + \vec{u}\,\mathrm{grad}(C_{\mathrm{v}}\overline{T}) + \mathrm{div}(C_{\mathrm{v}}\theta\vec{u} - C_{\mathrm{v}}\overline{\theta\vec{u}}) + R\theta\vec{\nabla}\cdot\overline{\vec{U}} + \overline{\mathrm{U}}(\vec{\nabla}\cdot\vec{q} - \mathrm{Rad}') = \frac{\vartheta}{\overline{\mathrm{U}}}(\mathrm{D}_\theta + R\overline{T}\vec{\nabla}\cdot\overline{\vec{U}} - \overline{\Pi}_\theta) + \Pi_\theta - \overline{\Pi}_\theta \tag{5b}$$

with

$$\mathrm{D}_\theta = \frac{\mathrm{D}(C_{\mathrm{v}}\overline{T})}{\mathrm{D}t} + \vec{\nabla}\cdot(C_{\mathrm{v}}\overline{\theta\vec{u}}).$$

3.5.3. Divergence of velocity

The relationships below were subsequently inferred from the transport equation of internal energy:

$$\vec{\nabla}\cdot\vec{U} = -\frac{1}{R\overline{T}}\left(\frac{\mathrm{d}(C_{\mathrm{v}}T)}{\mathrm{d}t} + R(\theta\vec{\nabla}\cdot\vec{U}) + \mathrm{U}(\vec{\nabla}\cdot\vec{Q} - \mathrm{Rad} - \Phi)\right), \tag{6a}$$

[2] Note that the internal mean fields can still be described in reference to the external ambient fluid (non-adiabatic) which can in turn be interpreted as a deviation from an aerostatic and adiabatic situation.

$$\vec{\nabla}\cdot\overline{\vec{U}} = -\frac{1}{R\overline{T}}\left(\frac{\mathrm{D}(C_v\overline{T})}{\mathrm{D}t} + \vec{\nabla}\cdot(C_v\overline{\theta\vec{u}}) + (R - C_v) \times \overline{\theta\vec{\nabla}\cdot\vec{u}} + \overline{\mathsf{U}(\vec{\nabla}\cdot\vec{Q} - \mathrm{Rad} - \Phi)}\right), \quad (6b)$$

$$\vec{\nabla}\cdot\vec{u} = -\frac{1}{R\overline{T}}\Big[C_v\frac{\mathrm{D}\theta}{\mathrm{D}t} + \vec{\nabla}\cdot(C_v\theta\vec{u} - C_v\overline{\theta\vec{u}}) + (R - C_v)(\theta\vec{\nabla}\cdot\vec{u} - \overline{\theta\vec{\nabla}\cdot\vec{u}}) + \overline{\mathsf{U}}(\vec{\nabla}\cdot\vec{q} - \mathrm{Rad}') + \vartheta(\vec{\nabla}\cdot\overline{\vec{Q}} - \overline{\mathrm{Rad}} - \Phi) + \vartheta(\vec{\nabla}\cdot\vec{q} - \mathrm{Rad}') - \overline{\vartheta(\vec{\nabla}\cdot\vec{q} - \mathrm{Rad}')}\Big]. \quad (6c)$$

3.6. Practical considerations

So far, no assumption has been made concerning the nature of the fluid. Therefore, the approach above is of a very general scope, and Eq. (1) identically applies to compressible gas or liquids. We shall now introduce the simplifying considerations used in the present experimental work.

Air was considered a non-chemically reacting ideal gas. In turbulent free convection, it can safely be assumed that $p/\overline{P}$, the ratio of pressure fluctuations to the absolute mean pressure, is negligible compared to unity. Thus, from a first-order Taylor expansion series of pressure, air specific volume can be linearly related to the temperature:

$$\mathsf{U} = \frac{1}{\rho} = \frac{RT}{P} \cong \frac{RT}{\overline{P}}\left(1 - \frac{p}{\overline{P}}\right) \cong \frac{RT}{\overline{P}}, \quad (7a)$$

from what we subsequently infer using Reynolds statistical decomposition:

$$\mathsf{U} = \overline{\mathsf{U}} + \vartheta = \frac{R\overline{T}}{\overline{P}} + \frac{R\theta}{\overline{P}}. \quad (7b)$$

Thus, in the transport equations, all correlations with the fluctuation of the inverse of density could be replaced with correlations with the temperature fluctuation, which are measurable.

Air was considered to be a Newtonian viscous fluid. All correlations with fluctuations of the kinematic viscosity ν and the thermal diffusivity α were neglected in the balance equations. However, the means of ν and α were allowed to vary with space consistently with the temperature range encountered in the experiments.

The Rossby number based on the height of the tunnel and the mean flow velocity was found to be much greater than unity so that it was assumed that body forces reduced to gravity.

Radiation and viscous dissipation of mechanical energy were neglected in the transport equation of internal energy.

4. Experimental results

Repeatable flow features were observed from a large number of experiments. It is the purpose of this paper to present and discuss the trends that emerged from a series of tests performed with stable meteorological conditions on sunny days. The analysis was performed by examining the streamwise evolution of horizontally averaged mean values of the statistical quantities involved in the balance equations, i.e., by looking at the flow as if it was perfectly homogeneous horizontally. The overbar used in the following, therefore, denotes both spatial and time averaging. The terms involving differentiation with respect to y and z thus were zero in average in all budget equations.

4.1. State of ambient fluid

Grid heating was started early in the morning and was maintained until the end of turbulence measurements in the tunnel. Approximately, 3 h of heating were needed for the ambient fluid, and thus the studied mean flow, to reach a constant or slightly evolutive temporal variation rate. The ambient fluid could be reasonably assumed to be at rest. The mean pressure differential between the building floor and the ceiling was measured at several locations in the laboratory. It was estimated to be about 0.1 Pa across the building depth. Considering that there were no reasons for the vertical profile of mean pressure to substantially depart from a linear law, it was assumed that the pressure field was aerostatically distributed in the laboratory hall. Even though the thermocouple records clearly indicated a decrease in the ambient mean temperature with the distance to the convection tower, horizontal gradients of mean temperature were negligible compared to the relevant vertical gradients so that the ambient field could safely be considered homogeneously distributed horizontally. The average vertical gradient of the ambient mean temperature was estimated to be about 1 K/m, representative of a stable stratification.

4.2. Bulk characteristics of the mean flow

The temporal evolution of the mean temperature field inside the tower was well correlated with that of the mean temperature field outside of the tunnel. The mean temperature vertical gradient was nearly constant within the investigated region of the tunnel, equal to −0.3 K/m, thus representative of an unstable thermal stratification.

Two distinct regions were identified in the flow. The frontier between these two regions varied from one experiment to another. It was located between 3 and 5 m downstream of the grid. Although a weak acceleration, the mean flow velocity remained nearly constant in the

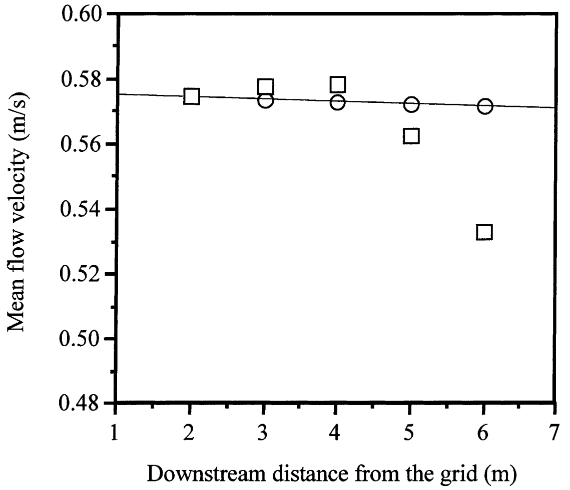

Fig. 2. Vertical profile of mean velocity longitudinal component: (□) present experiments; (○) what would have been obtained if $\vec{\nabla}\cdot\vec{U} = \frac{1}{\overline{T}}\frac{\mathrm{d}T}{\mathrm{d}t}$.

lower region. From 3 m downstream of the heating grid, a slight but noticeable decrease in the mean flow velocity was observed. The present mean flow velocity profile was compared to the relevant profile we would have obtained for an incompressible fluid flowing in an aerostatic and adiabatic atmosphere, taking for initial velocity the one measured at 2 m downstream of the grid in the present experiment (Fig. 2).

The unequivocal discrepancy between the two profiles puts forward the necessity to use a compressible form of the Navier–Stokes equations in the present case.

4.3. Balance of terms in the transport equation of internal energy

Prior to analysis, it must be noted that the term $R\overline{T}\vec{\nabla}\cdot\overline{\vec{U}}$ on the left-hand side of Eq. (5a) is to be considered an additional source/sink term due to variations of mean specific volume as a consequence of mass conservation. It does not appear in the transport equation of internal energy for an incompressible fluid. It actually replaces the pressure term present in the standard formulation of the transport equation of internal energy. It may be regarded as the homologue of the gravity term in the momentum equation. However, the role it plays in the transport equation of internal energy is not comparable to that played by the gravity term in the transport equation of mean momentum, as will be shown.

In Eq. (5a), all quantities except $(C_\mathrm{v} - R)\overline{\theta\vec{\nabla}\cdot\vec{u}}$ coming from $\overline{\Pi}_\theta$ were measured directly or estimated. Analysis of experimental results showed that the transport equation of mean internal energy finally reduced to Eq. (8) where we only kept the dominant terms and the unknown contribution of the non-measurable terms. The latter include the diffusion term and the dominant part of the deviation term:

$$\overline{\vec{U}}\nabla(\overline{T}) + \vec{\nabla}\cdot(\overline{\theta\vec{u}}) - \overline{\theta\vec{\nabla}\cdot\vec{u}} \cong -\frac{R}{C_\mathrm{v}}(\overline{T}\vec{\nabla}\cdot\overline{\vec{U}} + \overline{\theta\vec{\nabla}\cdot\vec{u}}). \tag{8}$$

Two cases may here be considered and examined separately:

- Diffusion reduces to $\partial(\overline{\theta u})/\partial x$ (the only component that could be measured). The value of $\partial(\overline{\theta u})/\partial x$ is much lower than transport by mean flow. Therefore, we are led to suggest

$$\overline{\theta\vec{\nabla}\cdot\vec{u}} \cong \left(\frac{\gamma-1}{\gamma-2}\right)\overline{T}\vec{\nabla}\cdot\overline{\vec{U}} - \left(\frac{1}{\gamma-2}\right)\overline{\vec{U}}\vec{\nabla}(\overline{T}). \tag{9}$$

  Due to the high value of $\vec{\nabla}\cdot\overline{\vec{U}}$ in the present case, volume fluctuation effects that reflects the correlation $\overline{\theta\,\mathrm{div}\,\vec{u}}$ turn out to play a key role as regards couplings between the temperature and velocity mean fields (Fig. 3).
- Making the hypothesis customarily used for this kind of flow, namely, that $\overline{T}\vec{\nabla}\cdot\overline{\vec{U}}$ is equal to zero, then the transport equation of mean internal energy can only be balanced if we admit that lateral diffusion $(\partial\overline{\theta v}/\partial y + \partial\overline{\theta w}/\partial z)$ is constrained by the semi-confinement of the investigated flow, and compensates for the effects of volume fluctuation.

In both cases, our results support earlier observations in turbulent free convection, and more precisely in plumes. They additionally tend to give evidence that

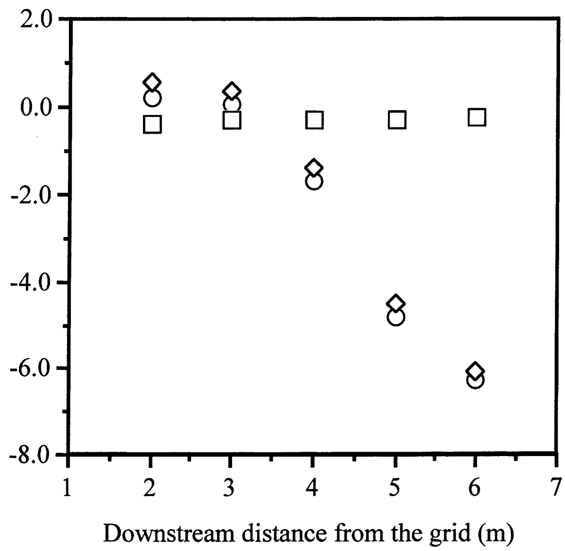

Fig. 3. Dominant terms in the transport equation of internal energy: (□) $-\frac{1}{(\gamma-2)}\overline{\vec{U}}\cdot\vec{\nabla}(\overline{T})$; (◇) $-\frac{(\gamma-1)}{(\gamma-2)}\overline{T}\vec{\nabla}\cdot\overline{\vec{U}}$; (○) $\overline{\theta\vec{\nabla}\cdot\vec{u}}$.z.

effects of volume variation contribute to the mechanisms of energy conservation. Therefore, the corresponding terms should be accounted for in the relevant balance equation independently of which one of the two hypotheses above actually applies.

Furthermore, experimental results seem to indicate that here effects of volume fluctuation tend to offset the effects of mean volume variation. This would suggest that specific volume could not vary without fluctuating. For an incompressible fluid, these two contributions should compensate for each other exactly. In other words, one could reformulate the Boussinesq approximation by saying it consists in assuming that effects of mean volume variation and volume fluctuation cancel each other out exactly.

4.4. Balance of terms in the transport equation of mean momentum

Analysis of experimental results showed that the transport equation of mean momentum reduced to Eq. (10) below after projection along the x-axis. All viscous terms in $\vec{\overline{\Pi}}_u|_x$ including the term proportional to $\overline{\theta\vec{\nabla}\cdot\vec{u}}$ were either measured or estimated. They could be neglected in the balance compared to the known dominant terms of the momentum equation. Two unknown quantities remained in $\vec{\overline{\Pi}}_u|_x$. They were accounted for in the analysis for lack of information about their possible contribution to the balance:

$$\underbrace{\overline{U}\frac{\partial\overline{U}}{\partial x}+\frac{\partial\overline{uu}}{\partial x}}_{\vec{\overline{D}}_u|_x}+g+\underbrace{\frac{R\overline{T}}{\overline{P}}\frac{\partial\overline{P}}{\partial x}}_{\text{unknown}}=\underbrace{-\frac{R}{\overline{P}}\overline{\theta\frac{\partial p}{\partial x}}+\overline{u\vec{\nabla}\cdot\vec{u}}}_{\vec{\overline{\Pi}}_u|_x}. \tag{10}$$

The term $\vec{\overline{\Pi}}_u|_x$ was expressed as a function of measurable terms by using existing links between the statistical modes of Reynolds and Favre [4]. Its contribution to the balance of the transport equation of mean momentum was found negligible. The gravity term formed the dominant contribution to the balance equation of momentum where it actually compensated almost completely for the pressure term. Eq. (10) could therefore be written in the form of Eq. (11) in which we reintroduced the state of the ambient fluid as a reference state:

$$\vec{\overline{\mathrm{D}}}_u|_x-g\frac{(\overline{T}-\overline{T}_\mathrm{a})}{\overline{T}_\mathrm{a}}+\frac{R\overline{T}}{\overline{P}_\mathrm{a}}\frac{\partial(\overline{P}-\overline{P}_\mathrm{a})}{\partial x}\cong\vec{\overline{\Pi}}_u|_x\cong 0. \tag{11}$$

However, even though the deviation terms were found to play a minor, if not negligible, role in the balance of the transport equation of mean momentum, note that the individual contribution of $-(R/\overline{P})(\overline{\theta\partial p/\partial x})$ and $\overline{u\vec{\nabla}\cdot\vec{u}}$ in $\vec{\overline{\Pi}}_u|_x$ could not be estimated through the present approach. The hypothesis that these two terms may be significant individually cannot be turned down definitely. Therefore, it could only be concluded that the role played by volume fluctuation effects is transparent in the transport equation of mean momentum in the form it was treated here. We eventually ended up with the classical formulation of the transport equation of mean momentum obtained by application of the Boussinesq approximation, consistently with earlier observations in the atmosphere.

Finally, it could be verified that the vertical distribution of pressure inside the tunnel followed the law of hydrostatic.

4.5. Balance of terms in the transport equation of the variance of temperature

The transport equation of the variance of temperature was written as

$$\underbrace{\frac{\mathrm{D}(\overline{\theta\theta})}{\mathrm{D}t}}_{1}+\underbrace{2\frac{R}{C_\mathrm{v}}\overline{\theta\theta}\vec{\nabla}\cdot\vec{\overline{U}}}_{2}+\underbrace{2\frac{\overline{\mathrm{U}}}{C_\mathrm{v}}\overline{\theta\vec{\nabla}\cdot\vec{q}}}_{3}+\underbrace{2\vec{\nabla}(\overline{T})\cdot\overline{\theta\vec{u}}}_{4}$$
$$+\underbrace{\vec{\nabla}\cdot(\overline{\theta\theta\vec{u}})}_{5}=2\frac{\overline{\vartheta\theta}}{C_\mathrm{v}\overline{\mathrm{U}}}\left(\underbrace{\mathsf{D}_\theta}_{6}+\underbrace{R\overline{T}\vec{\nabla}\cdot\vec{\overline{U}}}_{7}-\underbrace{\overline{\Pi}_\theta}_{8}\right)$$
$$+\underbrace{\frac{2}{C_\mathrm{v}}\overline{\theta\Pi_\theta}}_{9}-\underbrace{\overline{\theta\theta\vec{\nabla}\cdot\vec{u}}}_{10}. \tag{12}$$

Prior to analysis, it is worth noting that Eq. (12) reveals supplementary terms of production of $\overline{\theta^2}$, namely, terms 6–8, in the form of self production/destruction through coupling with the volume fluctuation represented by $\overline{\vartheta\theta}$. Terms 2 and 7 on the left-hand side and right-hand side of Eq. (12), respectively, are identical. However, they do not account for the same physical mechanisms. Term 2 proceeds from a conversion of the pressure term. It represents production by interaction with variations of mean specific volume. This term is customarily neglected in the study of atmospheric flows where the hypothesis of quasi-incompressibility is associated with an assumption of quasi-nullity of the divergence of mean velocity. Furthermore, interestingly enough the sum of terms 6 –8 between brackets can easily be estimated as it actually stems from the transport equation of internal energy. In contrast, it would not be so easy to estimate term 8 individually when canceling out terms 2 and 7.

Experimental results showed that the sum of terms 6–8 was fully negligible in the balance. Thus, it is interesting to note that these terms play a role fundamentally different from that played by their counterpart, namely, the buoyancy term, in the balance of the variance of velocity fluctuations (see further).

After estimation of the dominant terms, Eq. (12) finally was reduced to

$$\underbrace{\overline{U}\frac{\partial(\overline{\theta\theta})}{\partial x}}_{\mathrm{a}}+\underbrace{2\frac{R}{C_{\mathrm{v}}}\overline{\theta\theta}\frac{\partial\overline{U}}{\partial x}}_{\mathrm{b}}+\underbrace{\gamma\varepsilon_{\theta}}_{\mathrm{c}}+\underbrace{2\frac{\partial\overline{T}}{\partial x}\overline{\theta u}}_{\mathrm{d}}+\underbrace{\frac{\partial(\overline{\theta\theta u})}{\partial x}}_{\mathrm{e}}$$

$$\cong\underbrace{(3-2\gamma)\overline{\theta\theta\vec{\nabla}\cdot\vec{u}}}_{\mathrm{f}}-\underbrace{2(\gamma-1)\overline{T}\overline{\theta\vec{\nabla}\cdot\vec{u}}}_{\mathrm{g}}. \tag{13}$$

We did not observe the streamwise decay of the variance of temperature typical of grid turbulence in forced convection. Instead, an increase in $\overline{\theta\theta}$ was noticed starting at 4 m downstream of the grid.

Fig. 4 shows that convection does not balance the sum (production − dissipation − diffusion). Therefore, it is necessary to account for the contribution of the deviation terms to close the budget. The contribution of the deviation was deduced as residual from the imbalance. This term is positive, which indicates that relevant effects contribute to the production of turbulent energy. Consistently enough, these effects are most significant at 2 m downstream of the grids in a region close to the origin of the flow.

Furthermore, results suggest that, whereas production of turbulent energy due to volume fluctuation is commonly considered to be of second order, it here prevails over 'classical' production by mean gradients (Fig. 5). This remains to some extent consistent with the approach chosen here since this flow was designed so as to minimize production by mean gradients compared to production by buoyancy effects.

It must finally be noticed that the order of magnitude of the overall contribution of the deviation terms is determined mainly by the rather large values of the dissipation term whose estimation must be done carefully, as was discussed earlier (Fig. 4).

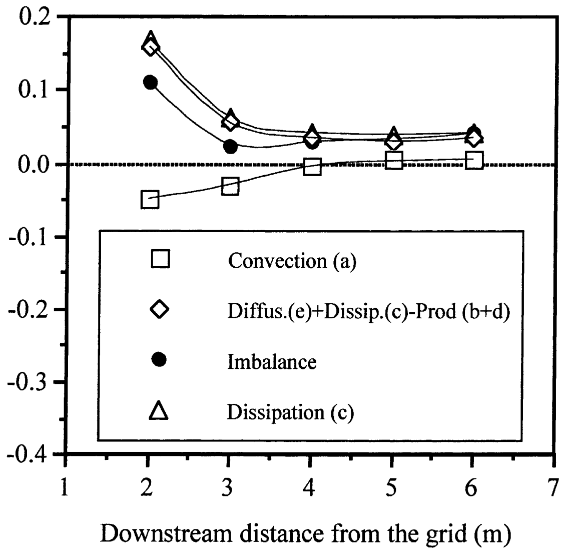

Fig. 4. Dominant terms in the transport equation of the temperature variance.

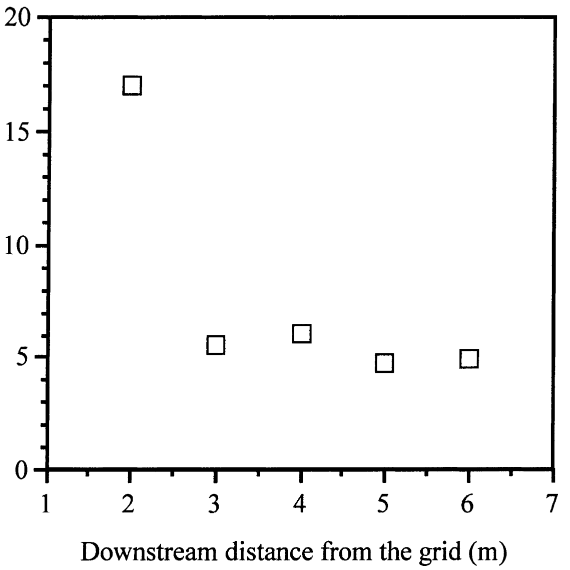

Fig. 5. Ratio of production of turbulent energy by volume fluctuation to 'classical' production by mean gradients:

$$(\square)\quad\frac{(3-2\gamma)\overline{\theta\theta\vec{\nabla}\cdot\vec{u}}-2(\gamma-1)\overline{T}\overline{\theta\vec{\nabla}\cdot\vec{u}}}{-2\frac{R}{C_{\mathrm{v}}}\overline{\theta\theta}\frac{\partial\overline{U}}{\partial x}-2\overline{\theta u}\frac{\partial\overline{T}}{\partial x}}.$$

4.6. Balance of terms in the transport equation of the variance of velocity fluctuations

The transport equation of the mean square fluctuation of velocity is given by Eq. (14) after projection along the x-direction:

$$\underbrace{\frac{\mathrm{D}\overline{uu}}{\mathrm{D}t}}_{1}-\underbrace{2\overline{\mathrm{U}}\overline{u\frac{\partial\sigma_{xk}}{\partial x_k}}}_{2}+\underbrace{\frac{\partial\overline{uuu_k}}{\partial x_k}}_{3}+\underbrace{2\frac{\partial\overline{U}}{\partial x_k}\overline{uu_k}}_{4}$$

$$=2\frac{\overline{\vartheta u}}{\overline{\mathrm{U}}}\left(\underbrace{\vec{\mathrm{D}}_u\big|_x}_{5}+\underbrace{g}_{6}-\underbrace{\vec{\Pi}_u\big|_x}_{7}\right)$$

$$-\underbrace{\overline{uu\vec{\nabla}\cdot\vec{u}}}_{8}+\underbrace{\overline{2u\vec{\Pi}_u\big|_x}}_{9}. \tag{14}$$

Recall that if Eq. (14) had been written in the framework of the Boussinesq approximation, the right-hand side would reduce to the buoyancy term $g\overline{\theta u}/T_0$ with T_0 a reference temperature, usually of an adiabatic atmosphere.

Experimental results put forward an increase in the variance of the velocity fluctuations with the distance from the grids in contrast to what is usually observed for grid turbulence in forced convection.

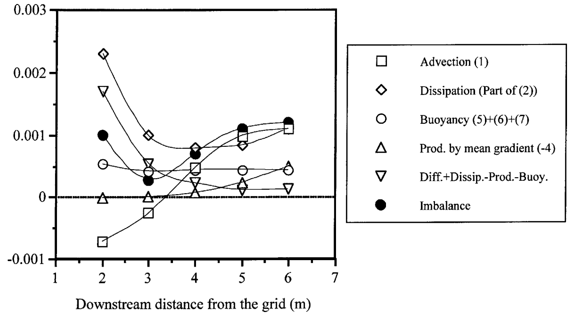

Fig. 6. Balance of terms in the transport equation of the velocity variance.

The sum of terms 5–7 was equal to g, as was shown in the analysis of the transport equation of mean momentum. Fig. 6 reveals that, as was expected, buoyancy effects form the major contribution to production of turbulent kinetic energy in the lower flow region where the flow is still extremely sensitive to initial conditions. Production by mean gradient increases with downwind distance from the grids, and is of the same order of magnitude as production by buoyancy at the highest measurement section. This is due mainly to the high values of the mean velocity gradient in the upper flow region, as a result of volume fluctuations by coupling with interactions at the upper flow boundaries, as was shown above.

Diffusion by turbulence (term 3) was constant across the investigated flow region. It was about 1–2 orders of magnitude smaller in order of magnitude than the advection term. This triple correlation in the end contributed little to the balance.

Dissipation ε_u was the dominant term. It did not balance the total production of turbulent energy by buoyancy and mean gradient in the lower region of the flow while dissipation and production were approximately evenly balanced from 4 m onwards as production by mean gradient increased. Therefore, the streamwise increase in the variance of the velocity fluctuation had to be related to additional mechanisms of production. Those could be due only to pressure and volume fluctuation effects (sum of terms 2, 8 and 9 in Eq. (14)). Analysis of the dominant terms showed that this sum reduced to

$$\text{term } 2 + \text{term } 8 + \text{term } 9 \cong -2\frac{R}{\overline{P}}\overline{\theta u\frac{\partial p}{\partial x}} + \overline{uu\vec{\nabla}\cdot\vec{u}} - 2\frac{R\overline{T}}{\overline{P}}\overline{u\frac{\partial p}{\partial x}}.$$

Interestingly enough, assuming $\overline{u\partial p/\partial x}$ forms the main contribution to $\overline{u_k\partial p/\partial x_k}$, it comes from the continuity equation (see [4])

$$\text{term } 2 + \text{term } 8 + \text{term } 9 \cong -2\frac{R}{\overline{P}}\overline{\theta u\frac{\partial p}{\partial x}} + \overline{uu\vec{\nabla}\cdot\vec{u}} + 2R\overline{T}\gamma\vec{\nabla}\cdot\vec{\overline{U}} + 2\frac{R\overline{T}}{\overline{P}}\overline{U}\frac{\partial\overline{P}}{\partial x}.$$

The relation above puts forward the origin of the high values of term 2 + term 8 + term 9, namely, the divergence of the mean velocity vector.

The pressure correlations in the sum above were modelled using the model of Launder et al. [5] for incompressible fluids extended by Ramirez Leon [6] and Ramirez Leon et al. [7] to quasi-compressible flows. After estimation of the dominant terms, Eq. (14) finally reduced to

$$\begin{aligned}\overline{U}\frac{\partial\overline{uu}}{\partial x} &= 2\frac{\overline{\theta u}}{\overline{T}}g - \frac{2}{3}\varepsilon_u - \frac{\partial\overline{uuu}}{\partial x} - 2\overline{uu}\frac{\partial\overline{U}}{\partial x}\\ &\quad+\left[-1,4\frac{\varepsilon_u}{\overline{\kappa^2}}\left(\overline{uu}-\frac{2}{3}\overline{\kappa^2}\right)+0,64\overline{\kappa^2}\frac{\partial\overline{U}}{\partial x}\right]\\ &\quad+\left[0,4\frac{\partial}{\partial x}(\overline{u_ku_ku}) - 0,4\overline{u_ku_ku}\frac{1}{\overline{T}}\frac{\partial\overline{T}}{\partial x}\right]\\ &\quad+\overline{uu\vec{\nabla}\cdot\vec{u}},\end{aligned} \tag{15}$$

where $\overline{uu\vec{\nabla}\cdot\vec{u}}$ remained the only unknown. The overall contribution of the last three terms on the right-hand side of Eq. (15) was inferred from the imbalance.

The pressure–diffusion was found negligible in the balance. The contribution of the pressure–strain term to the balance was significant close to the grids vanishing further downstream from the grid. Inversely, the contribution of volume fluctuation effects tended to prevail in the upper part of the flow (Fig. 7). Therefore, though

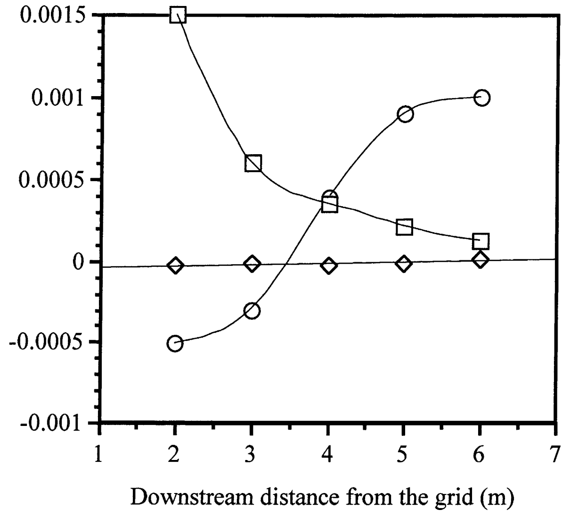

Fig. 7. Comparison of the contribution of pressure and volume fluctuation-related correlations in the balance of $\overline{uu}$: (□) pressure strain; (◇) pressure diffusion; (○) $\overline{uu\vec{\nabla}\cdot\vec{u}}$.

we could not quantify the individual contribution of $\overline{u\vec{\Pi}_u|_x}$ in Eq. (14), results suggest that volume and pressure fluctuation effects contribute equally to the balance of the velocity variance transport equation.

5. Conclusion

Experiments on free turbulent convection developing in an open vertical tunnel were conducted. Results were analyzed in the framework of a set of equations without too restrictive assumptions regarding specific volume variations. Effects of volume variation were investigated. Non-isovolume mechanisms were found to play a noticeable role in the flow configuration investigated here whereas they are considered to be second-order effects in most usual models. Relevant terms in the equations of turbulence appeared to be strongly related to the mean velocity divergence. Therefore, great care should be exercised in the experimental determination of this quantity. The internal energy equation could be balanced only by taking into consideration $\overline{\theta \operatorname{div} \vec{u}}$, the correlation between the temperature fluctuation and the divergence of the fluctuating velocity. However, volume variation and fluctuation effects compensate for each other so that their influence on the mean temperature field as well as on the mean velocity field is, in the end, weak. Although effects of volume variation actively contribute to the interactions between the mean temperature and velocity fields, thus determining the evolution and structure of these two fields, the mean flow seems to behave similarly to a situation of the Boussinesq type. Physical mechanisms associated with volume and pressure fluctuation seem to play a role of equal weight in the transport of turbulent kinetic energy. Therefore, this suggest that the physical analysis underlying commonly used closures where all couplings with the fluctuation of specific volume are ignored, if not neglected, should perhaps be revised. Experimental results tend to show as well that effects of volume fluctuation play a noticeable role with respect to the physical mechanisms driving the turbulent temperature field. The estimation of the relevant terms is shown to be strongly dependent of the estimation of ε_θ.

This work might sound frustrating since as a result of our inquiry we perhaps raised more questions than we have answered even though the results presented here remain consistent with earlier observations in the atmosphere. The work achieved here revives the debate about dilatation effects in turbulent free convection and, more generally, for weakly compressible flows. There is considerably greater need for additional data to verify the tentative conclusions we reached here. Accurate measurements of $(\vec{\nabla} \cdot \vec{U})$, among others, appear to pose serious experimental challenges. This point should deserve attention in future works. We hope finally that, in this work, one will find incentives to examination of the degree of applicability of customarily used models for turbulent free convection.

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